Soliton Switching in Fiber Coupler with Periodically Modulated Dispersion, Coupling Constant Dispersion and Cubic Quintic Nonlinearity

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In this paper we present the soliton dynamics in a two port fiber nonlinear directional coupler with periodically modulated dispersion and coupling constant dispersion that arose due to the intermodal dispersion between symmetric and antisymmetric modes of the coupler. The fiber material possessed cubic quintic nonlinearity. The influences of the coupling constant dispersion, periodically modulated dispersion and quintic nonlinearity on the soliton switching dynamics have been comprehensively studied. The expressions for the transmission coefficient, cross talk and extinction ratio have been derived in the context of both quintic nonlinearity and periodically modulated dispersion. It has been found that an increase in the value of the quintic nonlinearity has detrimental influence on the soliton switching. Our analytical results have been supported by numerical simulation.

Key words: Fiber Coupler; Periodically Modulated Dispersion; Coupling Constant Dispersion; Cubic Quintic Nonlinearity; Soliton Switching.

1. Introduction

Hasegawa and Tappert [1, 2] were the first theoretically predicting the existence of optical solitons in fibers, which was subsequently experimentally verified by Mollenauer et al. [3]. There has been tremendous interest in the investigation of optical solitons since then [1 – 5]. Since the last two decades, the nonlinear directional couplers (NLDCs) remain at the centre of research interest due to their technological applications to a good number of optical instrumentations like power splitting, wavelength division multiplexing-demultiplexing, polarization splitting, and fiber optic sensing. During this long time span, on one hand numerous theoretical as well as experimental research activities have been performed, on the other hand using these technologies many devices have been fabricated and employed for commercial use. In particular, the soliton switching in NLDC has been studied and reported comprehensively due to its technological relevance in all optical switching and signal processing [6 – 15]. Nowadays the NLDC is considered to be one of the vital building blocks of all-optical communication systems and signal processing devices. The pulse switching in an NLDC may be of two types. The first one is a power-controlled switching where the output is a function of the input power in one channel. Such power-dependent switching was first demonstrated by Wa et al. [15] in a GaAs/AlGaAs multiple quantum well (MQW). In the second type of switching, known as phase-controlled switching [16, 17], the phase difference between a weak and a strong input signal governs the switching dynamics, which may be considered as an attractive alternative of power-controlled switching. Apart from dual core coupler, all optical soliton switching in triple core fiber [18, 19] and multicore fiber [20] has also been studied with an anticipation to achieve sharper switching and comparatively richer switching dynamics. Initial investigations on NLDCs have been confined to a pair of symmetric waveguides separated by a constant distance and hence offering a constant coupling coefficient. Subsequently, different configurations with variable coupling coefficients have been studied in order to address more possibilities and flexibility in switching dynamics [21]. These nonconventional variants of the NLDC are broadly categorized into two classes. The first one is with a variable coupling coefficient, where the nonlinear coefficient remains fixed. Bent couplers [22] and X-junction [23] belong to this category. The other variant of an NLDC keeps the coupling constant fixed while a nonlinear coefficient varies with the propagation distance. Recently, a hybrid type NLDC has been reported, where both the coupling and nonlinear coefficients vary [22].

Although, most investigation on NLDCs considers each core as monomode waveguide, soliton formation is also possible for the more realistic case that includes two orthogonal polarizations in the coupler cores [24]. The study of soliton switching could not get a complete shape without the investigation on soliton interactions and stability analysis [25]. An NLDC consists of single mode fibers which are actually bimodal in nature that support two eigenmodes, i. e. symmetrical and antisymmetrical modes. This leads to intermodal dispersion (IMD), which can significantly change the switching dynamics in the NLDC [26]. The IMD can be captured via coupling constant dispersion in a coupled nonlinear Schrödinger equation, and soliton switching with coupling constant dispersion has been studied both numerically [27] and analytically [28].

Fibers of an NLDC may possess random fluctuation of their transverse dimension that causes birefringence and, hence, a periodically modulated dispersion (PMD) [23, 29]. Moreover, this transverse fluctuation is able to influence the nonlinear dispersion along the length of the fiber, which modifies the overall switching dynamics. The performance of a nonlinear directional fiber coupler with periodically modulated dispersion has been studied both analytically and numerically [30]. To the best of our knowledge, although soliton switching with the fiber coupler has been studied by separately taking into account the IMD and periodically modulated dispersion, no study has been reported on the combined effect of these two, which promises more interesting switching dynamics. Moreover, when the coupler is fabricated with semiconductor-doped fibers, Kerr nonlinearity alone will not be sufficient to describe the nonlinear dynamics of the system, as the higher order nonlinearity, namely cubic quintic nonlinearity, will come into play. Thus, in this paper we report the results of our extensive investigation on soliton switching in a fiber coupler with periodically modulated dispersion, coupling constant dispersion and cubic quintic nonlinearity. The organization of this paper is as follows. In Section 2, we will develop the necessary theoretical analysis required for the system. Section 3 contains the results and a discussion. A concluding remark is added in Section 4.

2. Mathematical Formulation

We consider the propagation of optical solitons of very short pulse width through an NLDC, fabricated with a dual core semiconductor-doped fiber having periodically modulated dispersion, coupling constant dispersion and cubic quintic nonlinearity. The evanescent-field coupling between the cores gives rise to linear coupling, whereas the cross phase modulation (XPM) is responsible for nonlinear coupling. The later one can be neglected, as the overlapping between the elemental modes corresponding to each core is comparatively small. Mathematically, this system can be presented [28, 30] by the following pair of coupled nonlinear Schrödinger equations (CNLSE), where the coupling is mediated by a linear coupling term:

$$\mathrm{i}\left(\frac{\partial u}{\partial \xi}+\delta\frac{\partial v}{\partial t}\right)+\frac{P(\xi)}{2}\frac{\partial^2 u}{\partial t^2}+|u|^2u+s|u|^4u=-k_0v, \tag{1}$$

$$\mathrm{i}\left(\frac{\partial v}{\partial \xi}+\delta\frac{\partial u}{\partial t}\right)+\frac{P(\xi)}{2}\frac{\partial^2 v}{\partial t^2}+|v|^2v+s|v|^4v=-k_0u, \tag{2}$$

$$P(\xi)=|\beta_2|\{1+A\cos(\omega\xi)\}, \tag{3}$$

where u and v are the normalized slowly varying envelope amplitude in the input core (core 1) and its neighbouring core (core 2), respectively, ξ is the normalized distance along the fiber length, t the normalized time, δ the first-order coupling constant dispersion coefficient, s the coefficient of quintic nonlinearity, which takes a negative sign in our present discussion as we are interested in self-defocusing quintic nonlinearity, k_0 is the normalized zeroth-order coupling coefficient, which is the measure of the strength of interaction between the fiber cores. k_0 depends on fiber characteristics, separation between the cores and operational frequency. $P(\xi)$ is the group velocity dispersion profile, and ω is the frequency of modulation in the periodically modulated dispersion fibers (PMDF). In order to explore the switching dynamics we solve the above CNLSE by employing the variational analysis method [31 – 35]. This method has been used successfully and extensively by several researchers to address different nonlinear optical problems, involving the nonlinear Schrödinger equation and its modified form. This formalism relies on the construction of a field Lagrangian for a suitable pulse profile with a number of slowly varying free parameters, which may describe the pulse amplitude, chirp, etc. One can increase the number of free parameters for a more accurate description of the physical phenomenon. This pulse profile is called the trial function. Since the variational method is an approximate method, the choice of a trial function is very crucial for the success of this method. For a choice close to a real one, this formalism can produce results with very good accuracy. With the help of the field Lagrangian and the trial function, one may obtain a set of ordinary differential equations (ODEs) for slowly varying free parameters. The main advantage of the variational method is its simplicity and capacity to provide a clear qualitative picture and a good quantitative result. This is the motivation for using this method in the present investigation.

The Lagrangian density for the system is given by

$$\begin{aligned} L = &\frac{\mathrm{i}}{2}\left(u\frac{\partial u^*}{\partial \xi} - u^*\frac{\partial u}{\partial \xi}\right) + \frac{P(\xi)}{2}\left|\frac{\partial u}{\partial t}\right|^2 - \frac{1}{2}|u|^4 \\ &-\frac{\gamma}{3}|u|^6 + \frac{\mathrm{i}}{2}\left(v\frac{\partial v^*}{\partial \xi} - v^*\frac{\partial v}{\partial \xi}\right) + \frac{P(\xi)}{2}\left|\frac{\partial v}{\partial t}\right|^2 - \frac{1}{2}|v|^4 \\ &-\frac{\gamma}{3}|v|^6 + \frac{\mathrm{i}\delta}{2}\left(v\frac{\partial u^*}{\partial t} - v^*\frac{\partial u}{\partial t} + v^*\frac{\partial v^*}{\partial t} - u^*\frac{\partial v}{\partial t}\right) \\ &-k_0(u^*v + uv^*). \end{aligned} \tag{4}$$

We assume the following ansatz that corresponds to a bright soliton solution in core 1 and 2, respectively:

$$\begin{aligned} u(t,\xi) = &A(\xi)\cos\{\theta(\xi)\}\,\mathrm{sech}\left\{\frac{t-\tau(\xi)}{a(\xi)}\right\} \\ &\cdot\exp\left\{-\mathrm{i}n(\xi)(t-\tau) + \mathrm{i}b(\xi)(t-\tau)^2 + \mathrm{i}\phi(\xi)\right\}, \end{aligned} \tag{5}$$

$$\begin{aligned} v(t,\xi) = &A(\xi)\sin\{\theta(\xi)\}\,\mathrm{sech}\left\{\frac{t-\tau(\xi)}{a(\xi)}\right\} \\ &\cdot\exp\left\{-\mathrm{i}n(\xi)(t-\tau) + \mathrm{i}b(\xi)(t-\tau)^2 - \mathrm{i}\phi(\xi)\right\}, \end{aligned} \tag{6}$$

where $A(\xi)$ is the amplitude of the pulse envelope, $\theta(\xi)$ the coupling angle, $\tau(\xi)$ the position of the pulse centre, $a(\xi)$ the pulse width, $b(\xi)$ represents a chirp, and $\phi(\xi)$ is the relative phase difference between the pulses. The averaged Lagrangian L_{av} of the system is obtained by the formula

$$L_{\mathrm{av}} = \int_{-\infty}^{\infty} L\mathrm{d}t. \tag{7}$$

Substituting (5) and (6) in (4), we get

$$\begin{aligned} L_{\mathrm{av}} = &\mathrm{i}a\left(A\frac{\partial A^*}{\partial \xi} - A^*\frac{\partial A}{\partial \xi}\right) + 2|A|^2 an\frac{\mathrm{d}\tau}{\mathrm{d}\xi} \\ &+ \frac{\pi^2}{6}|A|^2a^3\frac{\mathrm{d}b}{\mathrm{d}\xi} + 2|A|^2a\cos(2\theta)\frac{\mathrm{d}\phi}{\mathrm{d}\xi} \\ &+ \frac{|A|^2P}{3a} + |A|^2an^2P + \frac{\pi^2}{3}|A|^2a^3b^2P \\ &- \frac{2}{3}|A|^4a(\sin^4\theta + \cos^4\theta) \\ &- \frac{16\gamma}{45}|A|^6a(\sin^6\theta + \cos^6\theta) \\ &- 2|A|^2a(n\delta + k_0)\sin(2\theta)\cos(2\phi). \end{aligned} \tag{8}$$

Variations of the average Lagrangian with respect to different parameters, that characterize the system, give rise to the following set of evolution equations:

$$\frac{\mathrm{d}(2|A|^2a)}{\mathrm{d}\xi} = 0, \tag{9a}$$

$$\frac{\mathrm{d}n}{\mathrm{d}\xi} = 0, \tag{9b}$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}\xi} = \delta\sin(2\theta)\cos(2\phi) - nP, \tag{9c}$$

$$\frac{\mathrm{d}a}{\mathrm{d}\xi} = 2abP, \tag{9d}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\xi} = -(n\delta + k_0)\sin(2\phi), \tag{9e}$$

$$\begin{aligned} \frac{\mathrm{d}\phi}{\mathrm{d}\xi} = &-(n\delta + k_0)\cot(2\theta)\cos(2\phi) \\ &+ \left(\frac{E_0}{6a} + s\frac{E_0^2}{15a^2}\right)\cos(2\theta), \end{aligned} \tag{9f}$$

$$\begin{aligned} \frac{\mathrm{d}b}{\mathrm{d}\xi} = &\frac{2P}{\pi^2a^4} - 2b^2P - \frac{E_0}{\pi^2a^3}(\sin^4\theta + \cos^4\theta) \\ &- \frac{8E_0^2s}{15\pi^2a^4}(\sin^6\theta + \cos^6\theta). \end{aligned} \tag{9g}$$

Equation (9a) can be written as $2|A|^2a = E_0$ (constant). E_0 may be identified as the input soliton energy. The above equations are then used to get a comprehensive picture of the pulse evolution as well as the soliton switching in the coupler. A vital entity related to the switching feature in the NLDC is the fractional energy at the output end of the cores. Using (5) and (6), we can calculate the fractional energy at the output end of core 1 as

$$E_{1f} = \frac{\int_{-\infty}^{\infty} |u|^2 \mathrm{d}\tau}{\int_{-\infty}^{\infty} |u|^2 \mathrm{d}\tau + \int_{-\infty}^{\infty} |v|^2 \mathrm{d}\tau} = \cos^2(\theta). \quad (10)$$

Besides the coupling coefficient and the fractional energy, the other important parameters that control the coupler performance are the transmission coefficient, cross talk (X-talk) and extinction ratio (X-ratio). The cross talk may be defined as the ratio of power at an unwanted output port to that at a designated output port. In case of an on-off type switching the extinction ratio can be defined as the ratio of the output powers at "on" and "off" states. For a better switching performance the X-talk should be as low as possible, while the X-ratio should be as large as possible. Following the method as outlined by da Silva et al. [30], where the effective dispersion has been considered to be the average of the $P(\xi)$ profile of the fiber, we can analytically derive the energy transmission, X-talk and X-ratio.

The energy transmission through core 1 is given by

$$T_1 = \begin{cases} \frac{1}{2}\left[1 - \mathrm{cn}\left\{2k_0\xi, \left(\frac{A^2}{6k_0\bar{P}} + \frac{4A^4}{30k_0\bar{P}^2}\right)^2\right\}\right] & \text{for } \left(\frac{A^2}{6k_0\bar{P}} + \frac{4A^2}{30k_0\bar{P}^2}\right)^2 < 1, \\ \frac{1}{2}[1 - \mathrm{sech}(2k_0\xi)] & \text{for } \left(\frac{A^2}{6k_0\bar{P}} + \frac{4A^2}{30k_0\bar{P}^2}\right)^2 = 1, \end{cases} \quad (11)$$

where $\bar{P}$ is the average $P(\xi)$ profile and $\mathrm{cn}(x, y)$ is the Jacobian elliptic function. The cross talk turns out to be

$$\text{X-talk} = \begin{cases} 10\log_{10}\left(\frac{1}{2}\left[1 + \mathrm{cn}\left\{2k_0\xi, \left(\frac{A^2}{6k_0\bar{P}} + \frac{4A^2}{30k_0\bar{P}^2}\right)^2\right\}\right]\right) & \text{for } \left(\frac{A^2}{6k_0\bar{P}} + \frac{4A^2}{30k_0\bar{P}^2}\right)^2 < 1, \\ 10\log_{10}\left(\frac{1}{2}[1 + \mathrm{sech}(2k_0\xi)]\right) & \text{for } \left(\frac{A^2}{6k_0\bar{P}} + \frac{4A^2}{30k_0\bar{P}^2}\right)^2 = 1. \end{cases} \quad (12)$$

Similarly, the X-ratio has been calculated as

$$\text{X-ratio} = \begin{cases} 10\log_{10}\left(\frac{1 + \mathrm{cn}\left\{2k_0\xi, \left(\frac{A^2}{6k_0\bar{P}} + \frac{4A^2}{30k_0\bar{P}^2}\right)^2\right\}}{1 - \mathrm{cn}\left\{2k_0\xi, \left(\frac{A^2}{6k_0\bar{P}} + \frac{4A^2}{30k_0\bar{P}^2}\right)^2\right\}}\right) & \text{for } \left(\frac{A^2}{6k_0\bar{P}} + \frac{4A^2}{30k_0\bar{P}^2}\right)^2 < 1, \\ 10\log_{10}\left(\frac{1 + \mathrm{sech}(2k_0\xi)}{1 - \mathrm{sech}(2k_0\xi)}\right) & \text{for } \left(\frac{A^2}{6k_0\bar{P}} + \frac{4A^2}{30k_0\bar{P}^2}\right)^2 = 1. \end{cases} \quad (13)$$

3. Results and Discussion

Before discussing the switching characteristics, it will be useful to highlight the salient features of the first-order coupling constant dispersion coefficient δ. The value of δ can be calculated by the method employed by Ramos and Paiva [27]. For the sake of clarification, in Fig. 1 we redraw the variation of δ with the pulse width, which is more informative as we take k_0 as a parameter. This figure shows that, although for a comparatively larger pulse width ($\sim$ 1 ps), δ remains the same for all values of k_0; for a lower pulse width k_0 has significant influence on δ. For further study we collect the values of δ from Figure 1. In order to display the switching profile in the NLDC, (5) and (6) have been solved by directly using the split step Fourier method. Figure 2a shows the pulse profile in core 1 and core 2 in the presence of the PMD effect, whereas Fig. 2b shows the same without the PMD effect. In the present investigation, the total propagation distance has been taken to be equal to seven half beat lengths of the coupler. The fractional energy at the output end of core 1 depends on the input soliton energy E_0, δ, k_0 and the PMD effect. To perform a comparative study, in Fig. 3a the variation of E_{1f} with the normalized input soliton energy in the presence of the PMD effect is shown whereas Fig. 3b shows the same without the PMD effect. Both figures show that the quintic non-

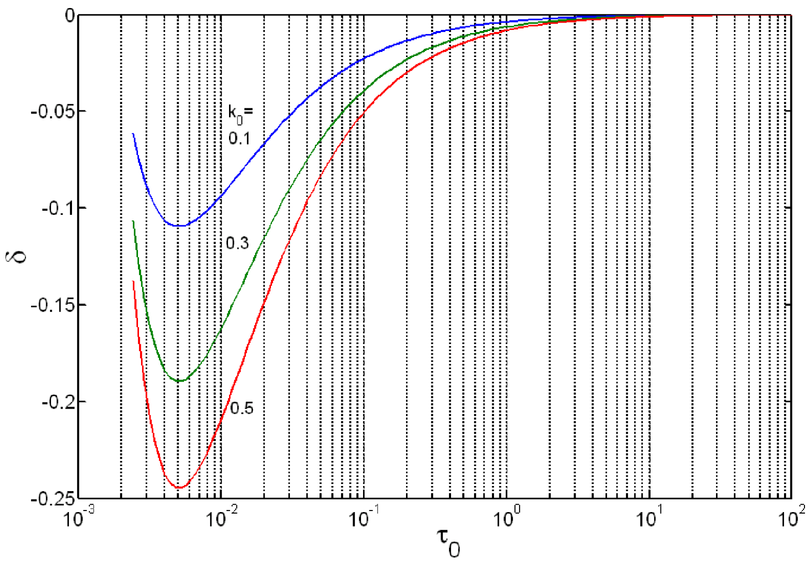

Fig. 1. Variation of the first-order coupling constant dispersion coefficient δ with soliton width τ_0 for different values of k_0.

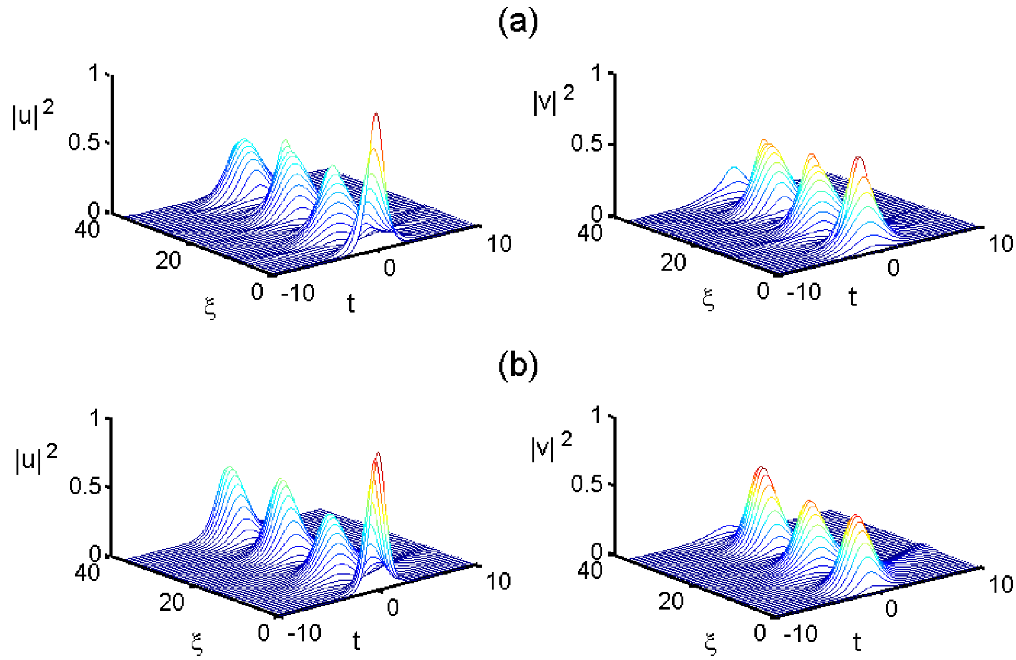

Fig. 2. Evolution of solitons u and v along the propagation length of the nonlinear directional coupler (NLDC), (a) with PMD effect ($\omega = 1$) and (b) without PMD effect; $k_0 = 0.3$, $\delta = -0.1456$, $s = -0.01$.

linearity has a rather detrimental effect on the switching characteristic. When $s = -0.01$, we get a very sharp switching characteristic for both cases, i. e. with and without PMD. We now look for the influence of the coupling coefficient on switching. Figure 4 depicts the variation of E_{1f} with normalized input soliton energy, considering k_0 as a parameter. From the figure it is found that, as k_0 increases, the critical energy for switching increases. This is shown in Fig. 5 for different values of the quintic nonlinearity.

4. Conclusion

We have presented the soliton dynamics in a two port fiber NLDC with periodically modulated dispersion and coupling constant dispersion, which arises

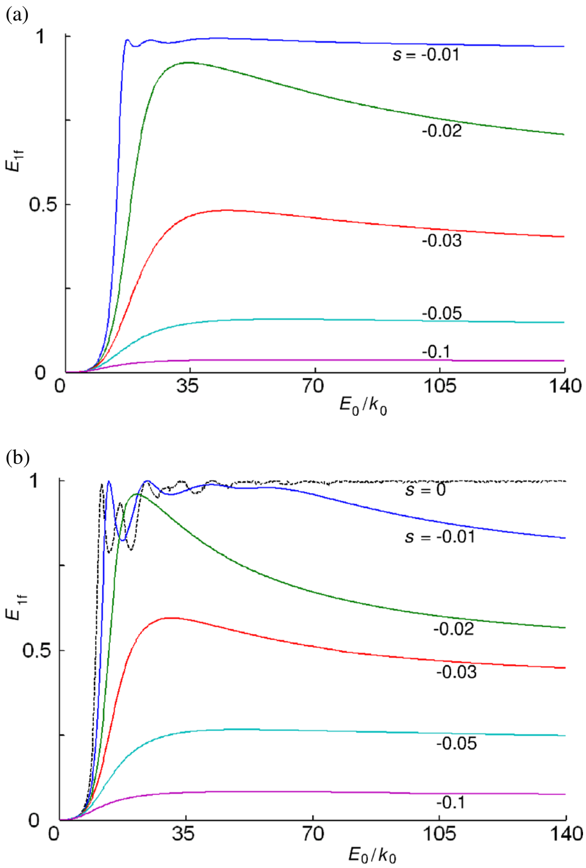

Fig. 3. Variation of the fractional energy at the output end of core 1 (i. e., $E_{1\mathrm{f}}$) with input soliton energy E_0, (a) with PMD effect ($\omega = 1$), (b) without PMD effect; $k_0 = 0.3$, $\delta = -0.1456$, $s = -0.01$.

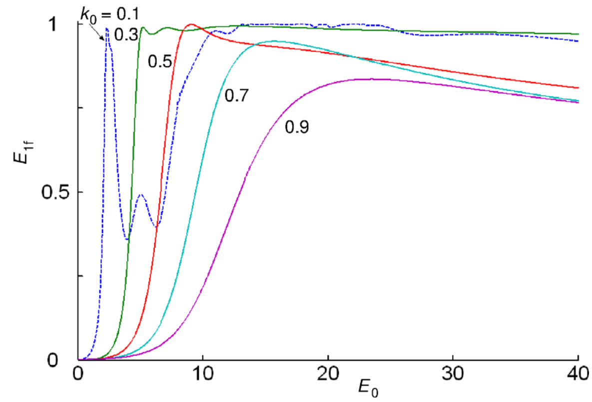

Fig. 4. Variation of $E_{1\mathrm{f}}$ with input soliton energy E_0. For different values of k_0, δ will acquire different values that can be obtained form Fig. 1; $s = -0.01$.

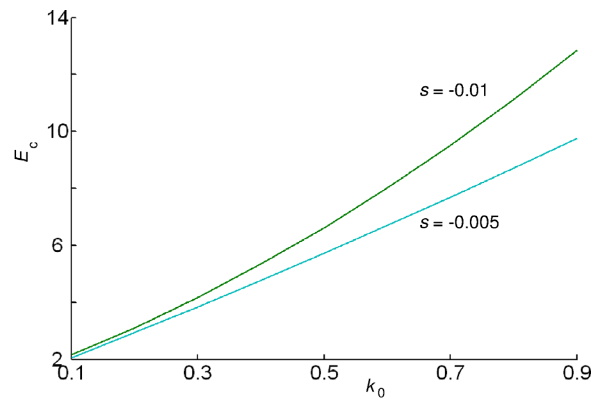

Fig. 5. Variation of the critical energy E_{c} with k_0 in the presence of PMD.

due to the intermodal dispersion between symmetric and antisymmetric modes of the coupler. The fiber material possessed cubic quintic nonlinearity. The effects of the coupling constant dispersion, periodically modulated dispersion and quintic nonlinearity on the switching dynamics have been comprehensively studied. The expressions for the transmission coefficient, cross talk and extinction ratio have been derived and discussed in the context of quintic nonlinearity as well as periodically modulated dispersion. It has been found that an increase in the value of the quintic nonlinearity has a detrimental influence on the soliton switching performance. Our analytical results have been supported by numerical simulations.

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